RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2023

SECOND YEAR [BATCH 2021-24]

Date : 27/05/2023 **PHYSICS [HONOURS]**

Time : 11 am – 1 pm **Paper : CC10** Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Consider the scattering of photon by an electron initially at rest. Show that the change in wavelength is equal to $\frac{h}{m_e c}(1-\cos\theta)$, where θ is the scattering angle, (other symbols have their usual meaning).

   Discuss the appearance of two peaks in the scattered spectra. (4+1)

   b) Show that the de-Broglie wavelength of an electron is equal to its Compton wavelength when its speed is $c/\sqrt{2}$. (2)

   c) An electron has a de-Broglie wavelength of $0.15\,\text{Å}$. Compute the phase and group velocities of that wave. (3)

2. Starting with $\left\langle \hat{X} \right\rangle = \int \psi^*(x,t)\,\hat{X}\,\psi(x,t)\,dx$

   a) Write the time dependent Schrodinger equation. (2)

   b) Using this show that (4)

   $$\frac{d\left\langle \hat{X} \right\rangle}{dt} = \int \psi^* \frac{i}{h}\left(\hat{H}\hat{X} - \hat{X}\hat{H}\right)\psi\, dx$$

   c) Given $\hat{H} = \frac{\hat{P}^2}{2m} + V\left(\hat{X}\right)$, find $\left[\hat{H}, \hat{X}\right]$. (2)

   d) Using this show $\frac{d\left\langle \hat{X} \right\rangle}{dt} = \frac{1}{m}\int \psi^*(x,t)\frac{d\psi(x,t)}{dx}\,dx$ (2)

3. a) Show that the momentum operator $\hat{P} = -i\hbar\frac{d}{dx}$ is Hermitian. (4)

   b) Given

   $$S_z = \frac{\hbar}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

   $$|\uparrow\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

   $$|\downarrow\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

   $$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$$

   Find $\langle S_z\rangle, \langle S_z^2\rangle, \langle \Delta S_z\rangle$. (2+2+2)

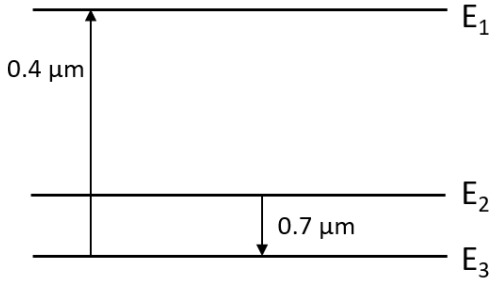
4. Two photons are entangled. One photon is with Virat (*V*) and another photon is with Gautam (*G*). Consider the entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_V |\downarrow\rangle_G + |\downarrow\rangle_V |\uparrow\rangle_G\right)$ where $|\uparrow\rangle, |\downarrow\rangle$ are eigenstates of S_z as mentioned in problem 3.

a) Virat measures the spin of photon and finds it to be $\left|\uparrow\right\rangle_V$. Then Gautam measures the spin of the photon with him. What are the possible outcome(s)? (2)

b) Are the photons still entangles after measurement? (2)

c) Again start with state $|\psi\rangle$. Now Gautam measures the spin of the photon in $|+\rangle, |-\rangle$ basis and finds to be $\left|-\right\rangle_G$. Then what is the state of the spin of the photon with Virat? (6)

5. a) Consider the energy level diagram (as shown in the figure below) of a typical three level ruby laser system with 1.6×10^{19} chromium ions per cubic centimeter. All the atoms by the 0.4 μm radiation decay rapidly to level E_2, which has lifetime $\tau = 3$ ms. Find the minimum pump power required (per cubic centimeter) to bring the system zero gain. (3)



b) Find the ratio of the rates of stimulated and spontaneous emissions at $T=10^3$ K for the visible radiation frequency 5×10^{14} Hz and microwave frequency 10^9 Hz. Comment on the result. (2)

c) Establish the relation between Einstein's A, B coefficients and hence comment on the incoherency observed in ordinary light. (4+1)

6. a) A radioactive element X decays to Y, which in turn decays to a stable element Z. The decay constant from X to Y is λ_1 and λ_2. At the beginning with, there only N_0 atoms of X. At short times ($t << 1/\lambda_1$ and as well as $1/\lambda_2$) find the number of Z. (3)

b) The Mass of hydrogen atom and of neutron are 1.008142 u and 1.008982 u. Calculate binding energy per nucleon and packing fraction of O^{16} nucleus. (2)

c) Mass of He nucleus, proton, and neutron in amu are 4.0026, 1.007895 and 1.008665. Find the energy required to knock out nucleon from He nucleus. (2)

d) Discuss the law of successive disintegration in the case of following transformation. (3)

$$P \xrightarrow{\lambda_1} Q \xrightarrow{\lambda_2} R$$

Where λ_1 and λ_2 are disintegration constant.

7. a) Which isobars of A=75 does the liquid drop model suggest to be the most stable nucleus? (3)

b) Establish the relation $A \approx 2Z$ for light light nuclei using semiempirical mass formula. Given; a_c= 0.71 MeV, a_n = 22.7 MeV, M($^1H_1$) =1.0078 u, M(n)=1.0086 u. (3)

c) Explain the shell model which predicts magic number. (4)

8. a) $_{28}Ni^{64}$ and $_{29}Cu^{64}$ have atomic masses 63.9280 u and 63.9298 u respectively. Which of them shows $\beta-$activity and of which type? Justify your answer. Given that $2m_e = 0.0011$ u. (3)

b) $_{90}Th^{232}$ $\alpha-$decays to its first daughter $_{88}Rs^{228}$. It is observed that a very thin foil containing 1 gm of $_{90}Th^{232}$ emits $\alpha-$particles from this at a decay rate of 4100/sec. Use this data to calculate the half life of $_{90}Th^{232}$. Also calculate the mean life. (3+1)

c) Explain the phenomenon of Pair Production. (3)

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